REALISTIC TOLERANCE ANALYSIS OF MICROWAVE NETWORKS

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ABSTRACT

A general formulation of the network tolerance problem is presented. It allows to deal, in a unified way, with all the disturbing effects, that may affect the network response. It can be used for the realistic analysis and design of networks. Possible simplifications of the formulation are discussed.

Introduction

Due to various disturbing effects the response of a microwave network can differ considerably from that predicted by a theoretical analysis.
The physical parameters of the network (e.g. dimensions, properties of materials) can be different from their nominal values, due to physical tolerances. Statistically dependent variations of the physical parameters may occur, caused e.g. by disturbing effects in the production process, affecting several parameters simultaneously.
To calculate the network response, a model of the physical structure is needed. The model parameters are affected by model uncertainties, an exact model usually not being available.
Finally, the practical source and load are different from those assumed in the design. This amounts to the introduction of external disturbing effects, such as the mismatches of source and load, and uncertainties on the position of the reference planes.
Any realistic design method should take into account the response variations caused by all these disturbing effects.
We shall present a formulation of the tolerance problem[1,2], which is general enough to deal with all disturbing effects in a unified way. This formulation is a generalization of that given by Bandler et al.[3-5]. It can be incorporated in known methods for worst-case analysis and design[1-7] as well as for statistical analysis and design[8-10].

General formulation of the tolerance problem

The disturbing effects will be represented by statistical variables μ^i_j, $j=1,2,\ldots,k_{\mu i}$, $i=1,2,\ldots,n$, or also μ_j, $j=1,2,\ldots,k_\mu$, where

$$k_\mu = \sum_{i=1}^{n} k_{\mu i} \tag{1}$$

We define

$$\underline{\mu}^i = \begin{bmatrix} \mu^i_1 \\ \vdots \\ \mu^i_{k_{\mu i}} \end{bmatrix}, \quad i = 1,\ldots n \;\; ; \quad \underline{\mu} = \begin{bmatrix} \underline{\mu}^1 \\ \vdots \\ \underline{\mu}^n \end{bmatrix} \tag{2}$$

We shall assume that the μ-variables are statistically independent. This implies that it should be possible to pinpoint the independent disturbing effects in the network.
The network parameters have nominal values p^{io}_j, $j=1,2,\ldots,k_{0i}$, and actual values p^i_j, $j=1,2,\ldots,k_i$, for $i=1,2,\ldots,n$. We define k_0, $\underline{p}^{io}$ and $\underline{p}^o$, as well as k, $\underline{p}^i$ and $\underline{p}$, analogously to k_μ, $\underline{\mu}^i$ and $\underline{\mu}$ in (1) and (2).
The following relationships between the statistical variables and the parameters are introduced :

$$\underline{p}^i(\underline{p}^{io},\underline{\mu}^i) \quad , \quad i = 1,2,\ldots,n \tag{3}$$

and

$$\underline{p}^{io}(\underline{p}^1,\underline{p}^2,\ldots,\underline{p}^{i-1}) \quad , \quad i = 2,3,\ldots,n \tag{4}$$

Usually, $\underline{p}^1$ contains the most fundamental parameters affected by independent statistical variations, while $\underline{p}^n$ contains the response functions (e.g. at k_n sample frequencies).
It is usually assumed that, for $j=1,\ldots,k_\mu$,

$$\mu^-_j \leqslant \mu_j \leqslant \mu^+_j \tag{5}$$

The region of all possible outcomes[1-6] is defined as

$$R_\mu = \{\underline{\mu} \mid \mu^-_j \leqslant \mu_j \leqslant \mu^+_j,\ j = 1,\ldots,k_\mu\} \tag{6}$$

The tolerance region[1-5] is defined in the parameterspace as

$$R_p = \{\underline{p}(\underline{\mu}) \mid \underline{\mu} \in R_\mu\} \tag{7}$$

R_μ is an orthotope, but R_p can have an arbitrary shape.
Similar regions can be defined in subspaces of the $\underline{\mu}$-or $\underline{p}$-space, e.g. :

$$R^i_\mu = \{\underline{\mu}^i \mid \underline{\mu} \in R_\mu\} \quad , \quad R^i_p = \{\underline{p}^i(\underline{\mu}) \mid \underline{\mu} \in R_\mu\} \tag{8}$$

Then

$$R^i_\mu = \mathcal{P}^{\mu i} R_\mu \quad , \quad R^i_p = \mathcal{P}^{p i} R_p \tag{9}$$

where $\mathcal{P}^{\mu i}$ and $\mathcal{P}^{p i}$ are projection operators.
Relationship (3) is often reduced to

$$p^i_j = p^{io}_j + \mu^i_j\ \delta^i_j \ , \text{ or } p^i_j = p^{io}_j(1 + \mu^i_j\ \delta^i_j) \tag{10}$$

with $\mu^+_j = -\mu^-_j = 1$.

δ^i_j is an absolute or relative tolerance.

Reduced formulation

If the complete formulation of a network problem should be too complicated for practical purposes, it can be simplified as follows. Take some $i > 1$, such that, for all $i < \ell \leqslant n$, (4) is reduced to

$$\underline{p}^{\ell o}(\underline{p}^i,\ldots,\underline{p}^{\ell-1}) \tag{11}$$

Let

$$p^i_{jM} = \underset{\underline{\mu}'^i \in R'^i_\mu}{\text{Max}}\ p^i_j(\underline{\mu}'^i), \quad p^i_{jm} = \underset{\underline{\mu}'^i \in R'^i_\mu}{\text{Min}}\ p^i_j(\underline{\mu}'^i) \tag{12}$$

for $j = 1,\ldots,k_i$, where

$$\underline{\mu}'^i = \begin{bmatrix} \underline{\mu}^1 \\ \vdots \\ \underline{\mu}^i \end{bmatrix} \quad , \quad R'^i_\mu = R^1_\mu \oplus R^2_\mu \oplus \ldots \oplus R^i_\mu \tag{13}$$

Define δ'^i_j and p'^{io}_j such that, for $j=1,\ldots,k_i$,

$$p^i_{jM} = p'^{i0}_j + \delta'^i_j \quad , \quad p^i_{jm} = p'^{i0}_j - \delta'^i_j \qquad (14)$$

The whole problem is then reformulated with a reduced number of variables $(k_i+k_{\mu,i+1}+\dots+k_{\mu n})$ and with parameters $\underline{p}^i, \underline{p}^{i+1},\dots,\underline{p}^n$.
The reduced formulation yields pessimistic results : the response variations predicted are generally larger than those given by the original formulation. The reason is that the fluctuations of $\underline{p}^i$ were originally statistically dependent, while they are now represented by statistically independent variables. In fact, R^i_p is replaced by the smallest orthotope containing it.

Applications

The worst-case for parameter p^i_j is defined as the solution of

$$\min_{\underline{\mu}\in R_\mu} p^i_j(\underline{\mu}) \qquad (15)$$

If p^i_j is a response function (i=n) we have the classical worst-case analysis problem[1,2,6]. Recently an efficient algorithm was proposed[2,6] for the solution of (15).
An accurate formulation may require a large k_μ, even for simple networks. The availability of an efficient algorithm is then a prerequisite for the practical usefulness of the formulation.

Examples

Example 1. Consider the one-section stripline transformer of fig. 1. Data are given in Table I.
It is assumed that the circuit is produced by cutting a mask, with dimensions W_i (i=1,2,3) and L, which is reduced by a factor K. The circuit is obtained by etching. Its nominal physical dimensions are given by

$$w^o_i = W_i\, K + E, \quad i = 1,2,3 \quad ; \ell^o = L\, K + E$$

where E is the "etching parameter" ($E^o = 0$).
W_i,L and E are affected by absolute tolerances, K by a relative tolerance. The physical parameters ε_r,b,t have relative tolerances.
From w_i,b and t, D^o_i is calculated, for i=1,2,3, by a formula given by Oliner and Altschuler[11]. These model parameters are affected by relative uncertainties. $Z_i = Z^o_i$ is then calculated from ε_r,b,t and D_i.
The total line length ℓ_t is nominally equal to ℓ, but is affected by a model uncertainty on d. The phase-angle $\beta=\beta^o$ follows immediately. X^o_i(i=1,2) is a function of $D_i,D_{i+1},\varepsilon_r$ and b[12], and has a relative uncertainty.

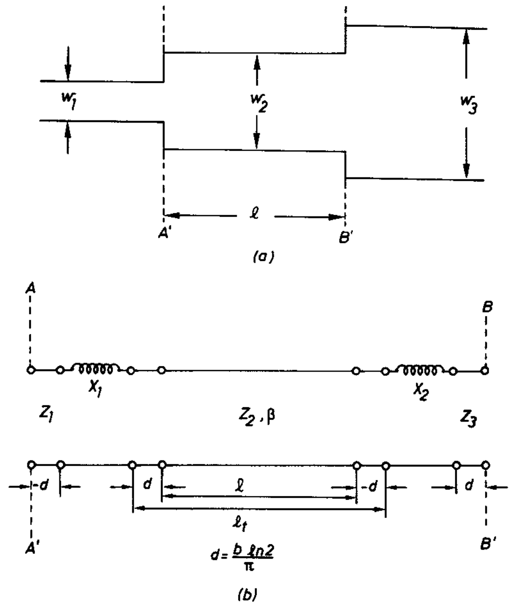

Fig. 1. (a) one-section stripline transformer
(b) equivalent circuit

The modulus of the input reflection coefficient, $|\rho| = |\rho|^o$, w.r.t. Z_1 and Z_3 is calculated from Z_1,Z_2,Z_3,β,X_1 and X_2. If a worst-case analysis should find the upper bound of the input reflection coefficient, the effect of the assumed mismatches and arbitrariness of the reference planes is eliminated by using explicit formulas[13]. These give the upper bound $|\rho_{in}|^o$ of the input reflection coefficient, w.r.t. Z_S and Z_L, as a function of $|\rho|,Z_1,Z_3$.
The response function to be used in (15) is then, with a specification S,

$$g = S - |\rho_{in}|^o$$

We take

$$\underline{p}^{1T} = [W_1\ W_2\ W_3\ L\ K\ E] \ ,$$

$$\underline{p}^{2T} = [w_1\ w_2\ w_3\ \ell\ \varepsilon_r\ b\ t]$$

$$\underline{p}^{3T} = [D_1\ D_2\ D_3\ \ell_t] \ , \ \underline{p}^{4T} = [Z_1\ Z_2\ Z_3\ \beta\ X_1\ X_2]$$

$$p^5 = |\rho| , \quad p^{60} = |\rho_{in}|^o , \ p^6 = g$$

There are 15 μ-variables.
Fig. 2 shows the result of a worst-case analysis, both using the complete formulation (a) and simplified formulations, obtained either by a reduction or by neglecting certain disturbing effects.

TABLE I. DATA FOR EXAMPLES

		Example 1	Example 2
nominal impedance generator, load		50.Ω , 20.Ω	
mismatch (max.modulus of reflection coefficient)	generator	0.025	
	load	0.025	
dimensions of mask	W_1	92.48 ± 0.1 mm	
	W_2,W	178.20 ± 0.1 mm	91.28 ± 1. mm
	W_3	308.84 ± 0.1 mm	
	L	168.74 ± 0.1 mm	188.24 ± 1. mm
reduction factor	K	0.05 ± 0.5 %	0.05 ± 20. %
etching tolerance		0.02 mm	0.05 mm
dielectric constant substrate	ε_r	2.54 + 1. %	2.54 ± 20. %
substrate thickness	b	6.35 mm ± 1. %	6.35 mm ± 20. %
strip thickness	t	0.051 mm ± 5. %	0.051 mm ± 20. %
uncertainty on effective line width	D_1,D_2,D_3,D	1. %	3. %
total line length	ℓ_t	0.4 mm	
parasitic reactance	X_1,X_2	3. %	

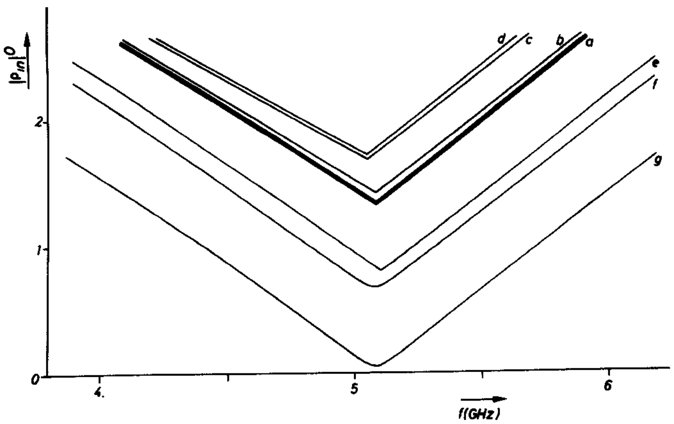

Fig. 2. Upper bound of input reflection coefficient of one-section stripline transformer, (a) general formulation; (b) formulation reduced with i=2; (c) formulation reduced with i=4; (d) formulation reduced with i=2 and i=4; (e) general formulation without model uncertainties; (f) idem, without model uncertainties and without tolerances; (g) nominal input reflection coefficient w.r.t. Z_1 and Z_3.

Example 2. Consider a single stripline section (width w, length ℓ). We use the same relations as in example 1 and assume uncertainties on W,L,K,E,ε_r,b,t and D. Let

$$\underline{p}^{1T} = [W\ L\ K\ E] \ , \quad \underline{p}^{2T} = [w\ \ell\ \varepsilon_r\ b\ t] \ ,$$
$$\underline{p}^{3T} = [D] \ , \quad \underline{p}^{4T} = [Z\ \tau]$$

where τ is the delay time. Relevant data are given in Table I. Fig. 3 shows the tolerance region R_p^4 in different situations. For cases c and d, the 4 uncertainties on $\underline{p}^1$ were reduced to 2 uncertainties on w and ℓ. The effect of a reduction with i=4 is illustrated by the circumscribing orthotope $R_p'^4$ (case a).

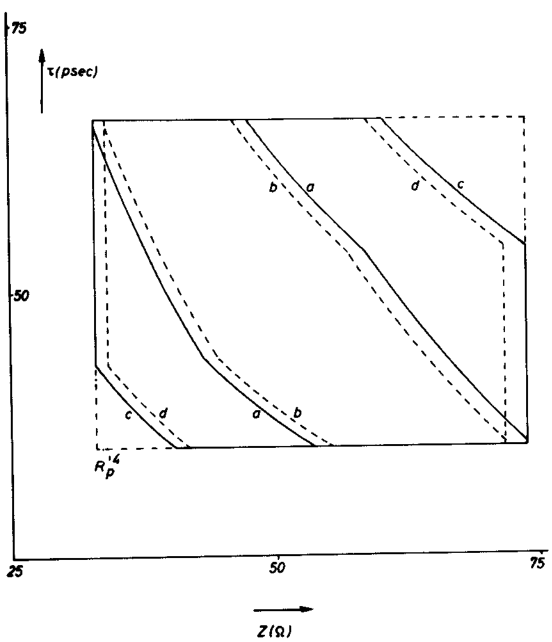

Fig. 3. Tolerance region R_p^4 in example 2; (a) complete formulation; (b) idem, without model uncertainty on D; (c) formulation reduced with i=2; (d) idem, without model uncertainty on D.

Conclusion

The unified formulation of the network tolerance problem allows to take into account all effects that can disturb the response of microwave networks. It can be incorporated in existing methods for analysis and design, leading to realistic design procedures.

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